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SOME NEW RESULTS FOR FUZZY WEAKLY CONTRACTIVE MAPPINGS
DISA REZULTATE TE REJA PER FUNKSIONET FUZZY DOBESISHT KONTRAKTIVE

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PERMBLEDHJE
Koncepti i funksioneve fuzzy në hapësirat metrike është futur për herë të parë nga Heilpern në 1981. Alber dhe Guerr-Delabriere(1997), Rhoades(2001), Zhang dhe Song(2009), Azam dhe Beg(2009) studjuan një klasë më të gjerë se klasa e funksioneve fuzzy: funksionet fuzzy dobësisht kontraktive në hapësirat e plota metrike. Qëllimi kryesor i këtij punimi është të investigojë rreth funksioneve fuzzy dobësisht kontraktive në kuadrin e hapësirave kuasi-metrike Smyth-të plota, duke zbutur kushtet për funksionin φ si dhe duke përgjithësuar klasën e hapësirave në të cilën jepen këta funksione. Rrezultati kryesor: Në qoftë se (X,d) është hapëesirë kuasi-metrike Smyth e plotë, $\mathscr{W}(X)$ është nënkoleksioni i bashkësive fuzzy në (X,d) me prerje α kompakte në (X,d^S) dhe $T_1,T_2 : X \to \mathscr{W}(X)$ janë dy funksione fuzzy dobësisht kontraktive, atëherë T_1,T_2 kanë një pikë fikse të përbashkët, si dhe rrjedhimet që dalin nga ky rrezultat përgjithësojnë teorema të njohura për pikat fikse. Metoda e përafrimeve duke zbutur kushtet për funksionet në shqyrtim si dhe klasën e hapësirave në të cilën ata jepen në mënyrë që të sigurojë ekzistencën e pikës fikse është përdorur. Shtrirja e rezultateve të deritanishme në klasën e hapësirave Smyth-të plota me koleksionin $\mathscr{W}(X)$ bëhet për arsye se në praktikë ato ndeshen shpesh.

SUMMARY
The notion of fuzzy mappings in the settings of metric spaces was introduced by Heilpern (1981). Alber and Guerr-Delabriere(1997), Rhoades(2001), Zhang and Song(2009), Azam and Beg(2009) studied the more general class of these mappings; the weakly fuzzy contractive mappings in complete metric spaces. The main purpose of this work is to investigate the notion of fuzzy set-valued weakly contractive mappings in the settings of Smyth-complete quasi-metric space, weakened the assumptions for the φ function and generalized the class of spaces in which are done. The main result: If (X,d) be a Smyth-complete quasi-metric space, $\mathscr{W}(X)$ is the sub-collection of fuzzy sets in X with d^S-compact α-level and $T_1,T_2 : X \to \mathscr{W}(X)$ be a generalized fuzzy weakly contractive mapping then T_1,T_2 have a common fixed point, as well as the corollaries of this result generalizes the many known fixed point theorems. The method of successive approximations used to approximate the fixed points. The extension of present results in the settings of Smyth-complete metric spaces with $\mathscr{W}(X)$, is because events in this case are mostly fuzzy sets.
Key words: Fixed point, Fuzzy mapping, weakly contractive.

1. INTRODUCTION AND PRELIMINARIES

The concept of weakly contractive point-to-point mappings is introduced by Alber and Guerr-Delabriere [1] in the settings of Hilbert spaces. Rhoades [10] showed that most results of [1] are still true for any Banach space. Also Bae [4] obtain fixed point theorems of multi-valued weakly contractive mapping.Zhang and Song [11] proved a common fixed point theorem for a pair of generalized φ-weak contractions in complete metric space. Heilpern [5] first introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy mappings. Since then many fixed point theorems for fuzzy mappings have been obtained by many authors. Azam and Beg [3] have introduced the concept of fuzzy weakly contractive mappings and proved a very interesting common fixed point theorem for two fuzzy weakly contractive mappings. Bose and Roychowdhury considered such fuzzy mappings and its two generalized versions, and proved some fixed point theorems. This work extends and generalizes the recent results for fuzzy weakly contractive mappings.

A quasi-metric on a nonempty set X is a nonnegative real valued function d on $X \times X$ such that, for all $x, y, z \in X$:

(a) $d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$, and (b) $d(x,y) \le d(x,z) + d(z,y)$.

A pear (X,d) is called a quasi-metric space, if d is a quasi-metric on X.

Each quasi-metric d on X induces a topology $\tau(d)$ which has a base the family of all d-balls $B(x,\varepsilon)$ where $B(x,\varepsilon) = \{y \in X : d(x,y) < \varepsilon\}$.

If d is a quasi-metric on X, then the function d^{-1} defined on $X \times X$ by $d^{-1}(x,y) = d(y,x)$ is also a quasi-metric on X. By $d \wedge d^{-1}$ we denote $\min\{d, d^{-1}\}$ and also we denote d^s the metric on X by $d^s(x,y) = \max\{d(x,y), d(y,x)\}$, for all $x, y \in X$.

A sequence $(x_n)_{n \in N}$ in a quasi-metric space (X,d) is called left K-Cauchy [15] if for each $\varepsilon > 0$ there is a $k \in N$ such that $d(x_n, x_m) < \varepsilon$ for all $n, m \in N$ with $m \ge n \ge k$.

A quasi metric space (X,d) is said to be Smyth-complete [16] if each left K-Cauchy sequence in (X,d) converges in the metric space (X, d^s).

Let (X,d) be a quasi-metric space and let $\mathscr{K}_0^S(X)$ be the collection of all nonempty compact subset of the metric space (X, d^s). Then the Hansdorff distance H_d on $\mathscr{K}_0^S(X)$ is defined by

$$H_d(A,B) = \max\{\sup d(a,B) : a \in A, \sup d(A,b) : b \in B\}$$

$$\text{whenever } A, B \in \mathscr{K}_0^S(X).$$

A fuzzy set on X is an element of I^X where $I = [0,1]$. If A is a fuzzy set and $x \in X$, then the function value $A(x)$ is called the grade of membership of x in A. The collection of all fuzzy sets in X is denoted by $\mathscr{F}(X)$.

Let $A \in \mathscr{F}(X)$ and $\alpha \in [0,1]$. The $\alpha -$ level set of A, denoted by A_α, is defined by

$$A_\alpha = \{x : A(x) \ge \alpha\} \text{ if } \alpha \in (0,1],$$

$$A_0 = \overline{\{x : A(x) \ge 0\}},$$

where $\overline{B}$ denotes the closure of the set B.

Definition 2.1 [5] Let (X,d) be a quasi-metric space. The family $\mathscr{W}(X)$ of all fuzzy sets on (X,d) is defined by

$\mathscr{W}(X) = \{A \in I^X : A_\alpha$ is d^s-compact for each $\alpha \in [0,1]$ and $\sup\{A(x) : x \in X\} = 1\}$.

Definition 2.2 [6] Let $A, B \in \mathscr{W}(X)$. Then A is said to be more accurate than B, denoted by $A \subset B$, if and only if $A(x) \le B(x)$ for each $x \in X$.

Definition 2.3 [6] Let (X,d) be a quasi-metric space and let $A, B \in \mathscr{W}(X)$ and $\alpha \in [0,1]$. Then we define ,

$$p_\alpha(A,B) = \inf\{d(x,y) : x \in A_\alpha, y \in B_\alpha\} = d(A_\alpha, B_\alpha)$$

$$D_\alpha(A,B) = H_d(A_\alpha, B_\alpha)$$

$$p(A,B) = \sup\{p_\alpha(A,B) : \alpha \in [0,1]\}$$

$$D(A,B) = \sup\{D_\alpha(A,B) : \alpha \in [0,1]\}.$$

For $x \in X$ we write $p_\alpha(x,A)$ instead of $p_\alpha(\{x\},A)$. We note that p_α is a non-decreasing function of α and D is a metric on $\mathscr{W}(X)$.

Definition 2.4 [5] A fuzzy mapping on a quasi-metric space (X,d) is a function F defined on X, which satisfies the following two conditions:

(1) $F(x) \in \mathscr{W}(X)$ for all $x \in X$

(2) If $a, z \in X$ such that $(F(z))(a) = 1$ and $p(a, F(a)) = 0$, then $(F(a))(a) = 1$

Definition 2.5 A fuzzy mapping $T: X \to \mathscr{W}(X)$ on a quasi-metric space (X,d) is said to be weakly contractive if

$$D(Tx,Ty) \le d(x,y) - \varphi(d(x,y) \text{ for each } x,y \in X,$$

where $\varphi:[0,\infty) \to [0,\infty)$ is a continuous and non-decreasing function such that φ is positive on $[0,\infty)$, $\varphi(0)=0$ and $\lim_{t\to\infty} \varphi(t) = \infty$.

Lemma 2.6 [5] Let (X,d) be a quasi-metric space. Then, for each $A \in \mathscr{W}(X)$ there exists $p \in X$ such that $A(p)=1$.

Lemma 2.7 [5] Let (X,d) be a quasi-metric space and let $A,B \in \mathscr{W}(X)$ and $x \in A_1$. There exists $y \in B_1$ such that $d(x,y) \le D_1(A,B)$.

Lemma 2.8 [5] Let (X,d) be a quasi-metric space and let $A,B \in \mathscr{W}(X)$.
Then $p(A,B) = p_1(A,B)$

Lemma 2.9 [5] Let (X,d) be a quasi-metric space and let $A \in \mathscr{W}(X)$ and $y \in A_1$.Then $p(x,A) \le d(x,y)$ for each $x \in X$.

Lemma 2.10 [6] Let $x \in X$, $A \in \mathscr{W}(X)$ and $\{x\}$ be a fuzzy set with membership function equal to a characteristic function of the set $\{x\}$, then $\{x\} \subset A$ if and only if $p_\alpha(x,A)=0$ for each $\alpha \in [0,1]$.

Lemma 2.11 [6] Let $x,y \in X$ and $A \in \mathscr{W}(X)$.
Then $p_\alpha(x,A) \le d(x,y) + p_\alpha(y,A)$

Lemma 2.12 [6] If $\{x_0\} \subset A$ then $p_\alpha(x_0,B) \le D_\alpha(A,B)$ for each $B \in \mathscr{W}(X)$

Lemma 2.13 [9] Let A and B be nonempty compact subsets of a metric space (X,d). If $a \in A$ then there exists a $b \in B$ such that $d(a,b) \le H(A,B)$.

Lemma 2.14 [5] Let (X,d) be a quasi-metric space and let $A \in \mathscr{W}(X)$. If $p(x,A)=0$, then there is $y \in cl_{\tau(d^{-1})}\{x\}$ such that $A(y)=1$.

Definition 2.15 [5] We say that a fuzzy mapping F on a quasi-metric space (X,d) has a fixed point if there exists $a \in X$ such that $(F(a))(a)=1$.

Definition 2.16 Two fuzzy mappings $T_1,T_2 : X \to \mathscr{W}(X)$ on a quasi-metric space (X,d) are called generalized $\varphi-$weak contractive if exists a continuous map $\varphi:[0,\infty)\to[0,\infty)$ with $\varphi(0)=0$ and $\varphi(t)>0$ for all $t>0$ such that for all $x,y \in X$

$$D(T_1x,T_2y) \le M(x,y) - \varphi(M(x,y))$$

where

$$M(x,y) = \max\{d(x,y), p(x,T_1(x)), p(y,T_2(y)), [p(x,T_2(y)) + p(y,T_1(x))]/2\}.$$

3. COMMON FIXED POINT THEOREMS

The function $\varphi:[0,\infty)\to[0,\infty)$ is said to be lower semi-continuous (l.s.c.) in $x \in [0,\infty)$, if for any sequence $\{x_n\}$ with $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} \varphi(x_n) = r$, then $\varphi(x) \le r$.

Theorem 3.1 Let (X,d) be a complete linear metric space and $T_1,T_2 : X \to \mathscr{W}(X)$ be two fuzzy generalized $\varphi-$weak contractive mappings satisfying the following condition:

$$\psi(D(T_1x,T_2y)) \le \psi(M(x,y)) - \varphi(M(x,y)) \qquad (3.1)$$

for each $x,y \in X$ and $\varphi:[0,\infty)\to[0,\infty)$ is l.s.c. function with $\varphi(0)=0$ and $\varphi(t)>0$ for all $t>0$, $\psi:[0,\infty)\to[0,\infty)$ is monotonically increasing continuous function with $\psi(0)=0$ and $\psi(t)>0$ for all $t>0$. Then there exists a point $z \in X$ such that $\{z\} \subset T_1(z)$ and $\{z\} \subset T_2(z)$.

Proof. Let x_0 be an arbitrary point of X. By Lemma 2.10 there exists $x_1 \in X$ such that $\{x_1\} \subset T_1(x_0)$. Then by Lemmas 2.10 and 2.11 we can choose $x_2 \in X$ such that $\{x_2\} \subset T_2(x_1)$ and

$$d(x_1,x_2) \le H(T_1(x_0)_1, T_2(x_1)_1).$$

By the condition of Theorem 3.1 we have

$$d(x_1,x_2)\le D_1(T_1(x_0),T_2(x_1))\le D(T_1(x_0),T_2(x_1))$$

Continuing this process, we construct a sequence $\{x_n\}_{n\in N}$ such that having chosen $x_n\in X$, we obtain $x_{n+1}\in X$ such that $\{x_{2n+1}\}\subset T_1(x_{2n})$, $\{x_{2n+2}\}\subset T_2(x_{2n+1})$ and

$$\begin{aligned} d(x_{2n},x_{2n+1})&\le D_1(T_1(x_{2n-1}),T_2(x_{2n}))\le D(T_1(x_{2n-1}),T_2(x_{2n}))\\ d(x_{2n+1},x_{2n+2})&\le D_1(T_1(x_{2n}),T_2(x_{2n+1}))\le D(T_1(x_{2n}),T_2(x_{2n+1}))\end{aligned} \tag{3.2}$$

Hence by the given hypothesis and as ψ is monotonically increasing we have,

$$\begin{aligned}\psi(d(x_{2n},x_{2n+1}))&\le\psi(M(x_{2n-1},x_{2n}))-\varphi(M(x_{2n-1},x_{2n}))\le\psi(M(x_{2n-1},x_{2n}))\\ \psi(d(x_{2n+1},x_{2n+2}))&\le\psi(M(x_{2n},x_{2n+1}))-\varphi(M(x_{2n},x_{2n+1}))\le\psi(M(x_{2n},x_{2n+1}))\end{aligned} \tag{3.3}$$

where

$$\begin{aligned} M(x_{2n-1},x_{2n})&=\max\{d(x_{2n-1},x_{2n}),p(x_{2n-1},T_1(x_{2n-1})),p(x_{2n},T_2(x_{2n})),\\ &\quad [p(x_{2n-1},T_2(x_{2n}))+p(x_{2n},T_1(x_{2n-1}))]/2\}\\ &\le\max\{d(x_{2n-1},x_{2n}),d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1}),\\ &\quad [d(x_{2n-1},x_{2n+1})+0]/2\}\\ &=\max\{d(x_{2n-1},x_{2n}),d(x_{2n},x_{2n+1})\}\end{aligned} \tag{3.4}$$

So by (3.3)

$$M(x_{2n-1},x_{2n})\le d(x_{2n-1},x_{2n}) \tag{3.5}$$

Similarly

$$M(x_{2n},x_{2n+1})\le d(x_{2n},x_{2n+1}) \tag{3.6}$$

As ψ is monotonically increasing by (3.3), (3.4), (3.5) and (3.6) for $n\ge 0$ we have,

$$\begin{aligned} d(x_{2n},x_{2n+1})&\le M(x_{2n-1},x_{2n})\le d(x_{2n-1},x_{2n})\\ d(x_{2n+1},x_{2n+2})&\le M(x_{2n},x_{2n+1})\le d(x_{2n},x_{2n+1})\end{aligned} \tag{3.7}$$

which shows that the sequence of positive real numbers $\{d(x_n,x_{n+1})\}$ is monotone non-increasing and bounded below. So there exits $l\ge 0$ such that

$$\lim_{n\to\infty} d(x_n,x_{n+1})=\lim_{n\to\infty} M(x_n,x_{n+1})=l \tag{3.8}$$

Again by (3.3) and (3.7) for $n\ge 0$ we have,

$$\psi(d(x_{n+1},x_{n+2}))\le\psi(M(x_n,x_{n+1}))-\varphi(M(x_n,x_{n+1})). \tag{3.9}$$

Since ψ is continuous and φ is lower semi-continuous, taking $n\to\infty$ we have

$\psi(l)\le\psi(l)-\varphi(l)$ ($\varphi(l)\le\liminf_{n\to\infty}\varphi(M(x_{n-1},x_n))$)

which is contradiction.

Therefore $\lim_{n\to\infty} d(x_n,x_{n+1})=\lim_{n\to\infty} M(x_n,x_{n+1})=0$.

Next we show that $\{x_n\}$ is Cauchy.

Let

$C_n=\sup\{d(x_i,x_j):i,j\ge n\}$

Obviously $\{C_n\}$ is decreasing. So there exists $c\ge 0$ such that $\lim_{x\to\infty} C_n=c$.

For every $k\in N$, there exists $n(k),m(k)\in N$ such that $n(k),m(k)\ge k$ and

$$C_k - \frac{1}{k} \le d(x_{m(k)}, x_{n(k)}) \le C_k \qquad (3.10)$$

So

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = c .$$

By (3.3) and for every $k \in N$ we have

$$\psi(d(x_{m(k)+1}, x_{n(k)+1})) \le \psi(M(x_{m(k)}, x_{n(k)})) - \varphi(M(x_{m(k)}, x_{n(k)}) \qquad (3.11)$$

where

$$\begin{aligned} M(x_{m(k)}, x_{n(k)}) &= \max\{d(x_{m(k)}, x_{n(k)}), p(x_{m(k)}, T_1(x_{m(k)})), p(x_{n(k)}, T_2(x_{n(k)})), \\ &\quad [p(x_{m(k)}, T_2(x_{n(k)})) + p(x_{n(k)}, T_1(x_{m(k)}))]/2\} \\ &\le \max\{d(x_{m(k)}, x_{n(k)}), d(x_{m(k)}, x_{m(k)+1}), d(x_{n(k)}, x_{n(k)+1}), \\ &\quad [d(x_{m(k)}, x_{n(k)+1}) + d(x_{n(k)}, x_{m(k)+1})]/2\} \end{aligned} \qquad (3.12)$$

As $k \to \infty$ in inequality (3.11) we have $\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)}) = c$. Since ψ is continuous and φ is lower semi-continuous and (3.11) holds, we have $c \le c - \varphi(c)$. Hence $\varphi(c) = 0$ and so $c = 0$.

Therefore, $\{x_n\}$ is Cauchy sequence.

Since (X,d) is complete there exists $z \in X$ such that $\lim_{n\to\infty} x_n = z$.

By Lemmas 2.8 and 2.9 we have:

$$p(z, T_2(z)) \le d(z, x_{2n+1}) + D(T_1(x_{2n}), T_2(z)) \qquad (3.13)$$

On the other hand

$$\begin{aligned} M(x_{2n-1}, z) &\le \max\{d(x_{2n-1}, z), p(x_{2n-1}, T_1(x_{2n-1})), p(z, T_2(z)), \\ &\quad [p(x_{2n-1}, T_2(z)) + p(z, T_1(x_{2n-1}))]/2\} \\ &\le \max\{d(x_{2n-1}, z), d(x_{2n-1}, x_{2n}), p(z, T_2(z)), \\ &\quad [d(x_{2n-1}, z) + d(z, x_{2n})]/2\} \end{aligned} \qquad (3.14)$$

Taking $n \to \infty$ in (3.14) we have

$$\lim_{n\to\infty} M(x_{2n-1}, z) = p(z, T_2(z)) .$$

Also, taking $n \to \infty$ in (3.13) we have

$$p(z, T_2(z)) \le \lim_{n\to\infty} D(T_1(x_{2n}), T_2(z)) \qquad (3.15)$$

So, by (3.14), (3.15) and properties of ψ and φ we have

$$\begin{aligned} \psi(p(z, T_2(z))) &\le \lim_{n\to\infty} \psi(D(T_1(x_{2n}), T_2(z))) \\ &\le \lim_{n\to\infty} \psi(M(x_{2n}, z)) - \lim_{n\to\infty} \varphi(M(x_{2n}, z)) \\ &= \psi(p(z, T_2(z))) - \varphi(p(z, T_2(z))) \end{aligned}$$

So $\varphi(p(z, T_2(z))) = 0$ and $p(z, T_2(z)) = 0$. Hence $\{z\} \subset T_2(z)$. Similarly $\{z\} \subset T_1(z)$.

Corollary 3.2 Let (X,d) be a complete linear metric space and $T_1, T_2 : X \to \mathscr{W}(X)$ be a fuzzy generalized $\varphi-$ weak contractive mappings satisfying the following condition:

$$D(T_1x, T_2y) \le M(x,y) - \varphi(M(x,y))$$

for each $x, y \in X$ and $\varphi : [0,\infty) \to [0,\infty)$ is l.s.c function with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$. Then there exists a point $z \in X$ such that $\{z\} \subset T(z)$.

Proof. The proof of this corollary is similar to the Theorem 3.1, if we take $\psi(t) = t$, for all $t \in [0,\infty)$

4. CONCLUSIONS

Smyth-completeness provides an efficient tool to the development of a consistent and unified topological foundation of many spaces which appear in several fields of Theoretical Computer Science. For instance, in semantic of languages, the set of all finite and infinite words, on a finite alphabet, can be structured as a Smyth-complete quasi-metric space [19]. Also, the Scott-line topology to the study of denotational semantics of dataflow networks, can be considered as a Smyth-complete quasi-metric space in many cases.

In this paper we establish a common fixed point theorem for fuzzy weakly contractive mappings in Smyth-complete quasi-metric spaces, which extend and generalize various comparable results from the literature [2, 3, 7, 10].

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