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ON QUASI-HYPERIDEALS IN TERNARY SEMIHYPERGROUPS

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SUMMARY
This paper deals with a class of algebraic hyperstructures called ternary semihypergroups, which are a generalization of ternary semigroups. In this paper we introduce the notions of quasi-hyperideal and bi-hyperideal in ternary semihypergroups and some properties of these kind of hyperideals in ternary semihypergroups are investigated. We study the structure of quasi-hyperideals in ternary semihypergroup without zero and in particular, we introduce and study the minimal quasi-hyperideals. Also, we introduce the notions of prime, semiprime, strongly prime, irreducible and strongly irreducible bi-hyperideals in ternary semihypergroups with zero and some properties of them are investigated. The space of strongly prime bi-hyperideals is topologized. we characterize those ternary semihypergroups for which each bi-hyperideal is strongly irreducible and also those ternary semihypergroups in which each bi-hyperideal is strongly prime.
Key words: Ternary semihypergroup, hyperideal, quasi-hyperideal, bi-hyperideal.

1 INTRODUCTION AND PRELIMINARIES

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory etc. Ternary algebraic operations were considered in the 19th century by several mathematicians such as Cayley [1] who introduced the notion of "cubic matrix" which in turn was generalized by Kapranov, et al. in 1990 [9]. Ternary structures and their generalization, the so-called n-ary structures, raise certain hopes in view of their possible applications in physics and other sciences. The notion of an n-ary group was introduced in 1928 by W. Dörnte [6] (under inspiration of Emmy Noether). The idea of investigations of n-ary algebras, i.e., sets with one n-ary operation, seems to be going back to Kasner's lecture [8] at the 53rd annual meeting of the American Association of the Advancement of Science in 1904. Different applications of ternary structures in physics are described by R. Kerner in [10]. Ternary semigroups are universal algebras with one associative ternary operation. The theory of ternary algebraic system was introduced by D. H. Lehmer [11] in 1932. He investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. The notion of ternary semigroups was introduced by S. Banach (cf. [12]). He showed by an example that a ternary semigroup does not necessary reduce to an ordinary semigroup. In 1965, Sioson [15] studied ideal theory in ternary semigroups. He also introduced the notion of regular ternary semigroups and characterized them by using the notion of quasi-ideals. In 1995, Dixit and Dewan [5] introduced and studied some properties of ideals and quasi-(bi-)ideals in ternary semigroups. Hyperstructure theory was introduced in 1934, when F. Marty [13] defined hypergroups based on the notion of hyperoperation, began to analyze their properties and applied them to groups. In the following decades and nowadays, a number of different hyperstructures are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics by many mathematicians. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set.

Ternary semihypergroups are algebraic structures with one associative ternary hyperoperation and they are a particular case of an n-ary semihypergroup (n-semihypergroup) for $n = 3$. In [7], we have introduced and study the quasi-hyperideals in semihypergroups. Recently, we [14] introduced and studied some classes of hyperideals in ternary semihypergroups. In this paper we introduce the notions of quasi-hyperideal and bi-hyperideal in ternary semihypergroups and some properties of these kind of hyperideals in ternary semihypergroups are investigated. We study the structure of quasi-hyperideals in ternary semihypergroup without zero and in particular, we introduce and study the minimal quasi-hyperideals. Also, we introduce the notions of prime, semiprime, strongly prime, irreducible and strongly irreducible bi-hyperideals in ternary semihypergroups with zero and some properties of them are investigated. The space of strongly prime bi-hyperideals is topologized. we characterize those ternary semihypergroups for which each bi-hyperideal is strongly irreducible and also those ternary semihypergroups in which each bi-hyperideal is strongly prime.

Recall first the basic terms and definitions from the hyperstructure theory.

A map $\circ : H \times H \to P^*(H)$ is called *hyperoperation or join operation* on the set H, where H is a non-empty set and $P^*(H) = P(H) \setminus \{\varnothing\}$ denotes the set of all non-empty subsets of H. A *hyperstructure* is called the pair $(H,\circ)$ where $\circ$ is a hyperoperation on the set H. A hyperstructure $(H,\circ)$ is called a *semihypergroup* if for all $x,y,z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that $\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$. If $x \in H$ and A,B are non-empty subsets of H then

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\} \text{ and } x \circ B = \{x\} \circ B.$$

A non-empty subset B of a semihypergroup H is called a *sub-semihypergroup* of H if $B \circ B \subseteq B$ and H is called in this case *super-semihypergroup* of B. H is called a *hypergroup* if for all $a \in H$, $a \circ H = H \circ a = H$.

A map $f : H \times H \times H \to P^*(H)$ is called *ternary hyperoperation* on the set H, where H is a non-empty set and $P^*(H) = P(H) \setminus \{\varnothing\}$ denotes the set of all non-empty subsets of H. A *ternary hypergroupoid* is called the pair (H,f) where f is a ternary hyperoperation on the set H.

If A,B,C are non-empty subsets of H, then we define

$$f(A,B,C) = \bigcup_{a \in A, b \in B, c \in C} f(a,b,c).$$

A ternary hypergroupoid (H,f) is called a *ternary semihypergroup* if $\forall a_1, a_2, \ldots a_5 \in H$, we have

$$\begin{aligned} f(f(a_1,a_2,a_3),a_4,a_5) &= f(a_1,f(a_2,a_3,a_4),a_5) \\ &= f(a_1,a_2,f(a_3,a_4,a_5)) \end{aligned}. \quad (*)$$

Since the set $\{x\}$ can be identified with the element x, any ternary semigroup is a ternary semihypergroup. It is clear that due to associative law in ternary semihypergroup (H,f), for any elements $x_1, x_2, \ldots x_{2n+1} \in H$ and positive integers m,n with $m \le n$, one may write

$$\begin{aligned} f(x_1,x_2,\ldots x_{2n+1}) &= f(x_1,\ldots x_m, x_{m+1}, x_{m+2}, \ldots x_{2n+1}) = \\ &= f(x_1,\ldots f(f(x_m,x_{m+1},x_{m+2}),x_{m+3},x_{m+4}),\ldots x_{2n+1}). \end{aligned}$$

Let (H,f) be a ternary semihypergroup. Then H is called a *ternary hypergroup* if for all $a,b,c \in H$, there exist unique $x,y,z \in H$ such that:

$$c \in f(x,a,b) \cap f(a,y,b) \cap f(a,b,z).$$

Let (H,f) be a ternary semihypergroup and T a non-empty subset of H. Then T is called a *ternary subsemihypergroup* of H if and only if $f(T,T,T) \subseteq T$.

Different examples of ternary semihypergroups can be found in [2, 3, 4, 14].

A ternary semihypergroup (H,f) is said to have a *zero element* if there exist an element $0 \in H$ such that for all $a,b \in H, f(0,a,b) = f(a,0,b) = f(a,b,0) = \{0\}$.

Let (H,f) be a ternary semihypergroup. An element $e \in H$ is called *left identity* element of H if $\forall a \in H$, $f(e,a,a) = \{a\}$. An element $e \in H$ is called an *identity* element of H if $\forall a \in H$, $f(a,a,e) = f(e,a,a) = f(a,e,a) = \{a\}$. It is clear that $f(e,e,a) = f(e,a,e) = f(a,e,e) = \{a\}$.

A non-empty subset I of a ternary semihypergroup H is called a *left* (*right, lateral*) *hyperideal* of H if $f(H,H,I) \subseteq I (f(I,H,H) \subseteq I, f(H,I,H) \subseteq I)$.

A non-empty subset I of a ternary semihypergroup H is called a *hyperideal* of H if it is a left, right and lateral hyperideal of H. A non-emtpy subset I of a ternary semihypergroup H is called *two-sided hyperideal* of H if it is a left and right hyperideal of H.

Let (H,f) be a ternary semihypergroup. For every element $a \in H$, the left, right, lateral, two-sided and hyperideal generated by a are respectively given by

$$\langle a \rangle_l = \{a\} \cup f(H,H,a)$$

$$\langle a \rangle_r = \{a\} \cup f(a,H,H)$$

$$\langle a \rangle_m = \{a\} \cup f(H,a,H)$$

$$\langle a \rangle_t = \{a\} \cup f(H,H,a) \cup f(a,H,H) \cup f(H,H,a,H,H)$$

$$\langle a \rangle = \{a\} \cup f(H,H,a) \cup f(a,H,H) \cup f(H,a,H) \cup f(H,H,a,H,H)$$

A left hyperideal I of a ternary semihypergroup H is called *idempotent* if $f(I,I,I) = I$.

A ternary semihypergroup H is said to be *regular* if for each $a \in H$, there exists an element $x \in H$ such that $a \in f(a,x,a)$.

A ternary semihypergroup H is called regular if all of its elements are regular.

It is clear that every ternary hypergroup is a regular ternary semihypergroup.

In this paper we are concerning the ternary semihypergroup which would be denoted by (H,f) or for short by H. In sections 1, 2 and 3 we are concerning the ternary semihypergroup without 0 and it has at least one idempotent element.

2 ON QUASI-HYPERIDEALS OF TERNARY SEMIHYPERGROUPS

Definition 2.1 *Let* (H,f) *be a ternary semihypergroup and* Q *a subset of* H. *Then* Q *is called a quasi-hyperideal of* H *if and only if*

$$f(Q,H,H) \cap f(H,Q,H) \cap f(H,H,Q) \subseteq Q \text{ and}$$

$$f(Q,H,H) \cap f(H,H,Q,H,H) \cap f(H,H,Q) \subseteq Q.$$

Theorem 2.2 *Let* (H,f) *be a ternary semihypergroup and* Q *a subset of* H. Q *is a quasi-hyperideal of* H *if and only if* Q *is the intersection of a right, lateral and a left hyperideal of* H.

Proof. Let R be a right hyperideal, M a lateral hyperideal and L a left hyperideal of H such that $Q = R \cap M \cap L$. Then Q is a quasi-hyperideal. In fact, we have:

$$f((R \cap M \cap L),H,H) \cap f(H,(R \cap M \cap L),H) \cap f(H,H,(R \cap M \cap L)) \subseteq$$

$$\subseteq f(R,H,H) \cap f(H,M,H) \cap f(H,H,L) \subseteq$$
$$\subseteq R \cap M \cap L,$$

$$f((R \cap M \cap L),H,H) \cap f(H,H,(R \cap M \cap L),H,H) \cap f(H,H,(R \cap M \cap L))) \subseteq$$

$$\subseteq f(R,H,H) \cap f(H,H,M,H,H) \cap f(H,H,L) \subseteq$$
$$\subseteq R \cap M \cap L.$$

Conversely, let Q be a quasi-hyperideal of H. Then obviously, $Q \subseteq \langle Q \rangle_R \cap \langle Q \rangle_M \cap \langle Q \rangle_L$, where $\langle Q \rangle_R = \bigcup_{q \in Q} \langle q \rangle_R$, $\langle Q \rangle_M = \bigcup_{q \in Q} \langle q \rangle_M$ and $\langle Q \rangle_L = \bigcup_{q \in Q} \langle q \rangle_L$. Moreover,

$$\langle Q \rangle_R \cap \langle Q \rangle)_M \cap \langle Q \rangle_L$$
$$= (Q \cup f(Q,H,H)) \cap (Q \cup f(H,Q,H) \cup f(H,H,Q,H,H)) \cap$$

$$\cap (Q \cup f(H,H,Q)) =$$

$$= Q \cup (f(Q,H,H) \cap ((f(H,Q,H) \cup f(H,H,Q,H,H)) \cap$$
$$\cap f(H,H,Q)) \subseteq Q.$$

Whence, $Q = \langle Q \rangle_R \cap \langle Q \rangle_M \cap \langle Q \rangle_L$.

Corollary 2.3 *Every quasi-hyperideal of a ternary semihypergroup* H *is a ternary subsemihypergroup.*

Proof. If Q is a quasi-hyperideal of H, then we have

$$f(Q,Q,Q) \subseteq f(Q,H,H) \cap (f(H,Q,H) \cup f(H,H,Q,H,H)) \cap f(H,H,Q) \subseteq Q.$$

Lemma 2.4 *Every left, right and lateral hyperideal of a ternary semihypergroup* H *is a quasi-hyperideal of* H.

Remark 2.5 *The converse of Lemma 2.4 is not true, in general, that is, a quasi-hyperideal may not be a left, right or a lateral hyperideal of* H.

Proposition 2.6 *If* Q *is a quasi-hyperideal of a ternary semihypergroup* (H,f) *and* T *is a ternary subsemihypergroup of* H, *then* $Q \cap T$ *is a quasi-hyperideal of* H.

Lemma 2.7 *The intersection of arbitrary collection of quasi-hyperideals of a ternary semihypergroup* (H,f) *is a quasi-hyperideal of* H.

Proof. Let $Q_i (i \in I)$ be any collection of quasi-hyperideals of H and let $\bigcap_{i\in I} Q_i$ be the intersection of them. This is indeed a quasi-hyperideal. In fact, if it is empty, the result is obvious. In general, we have for all $i \in I$:

$$f((\bigcap_{i\in I} Q_i),H,H) \cap (f(H,(\bigcap_{i\in I} Q_i),H) \cup f(H,H,(\bigcap_{i\in I} Q_i),H,H)) \cap$$
$$\cap f(H,H,(\bigcap_{i\in I} Q_i)) \subseteq f(Q_i,H,H) \cap (f(H,Q_i,H) \cup f(H,H,Q_i,H,H)) \cap$$
$$f(H,H,Q_i) \subseteq Q_i.$$

Hence

$$f((\bigcap_{i\in I} Q_i),H,H) \cap (f(H,(\bigcap_{i\in I} Q_i),H) \cup f(H,H,(\bigcap_{i\in I} Q_i),H,H)) \cap$$
$$f(H,H,(\bigcap_{i\in I} Q_i)) \subseteq \bigcap_{i\in I} Q_i.$$

Theorem 2.8 *The collection* L *of all quasi-hyperideals of a ternary semihypergroup* (H,f) *is a complete hyperlattice.*

Proof. It is clear that L is partially ordered by inclusion. The infimum of any collection of quasi-hyperideals $Q_i (i \in I)$ is obvious the $\bigcap_{i\in I} Q_i$ by Lemma 2.8.

Similarly, we set

$$\vee_{i\in I} Q_i = \left\langle \bigcup_{i\in I} Q_i \right\rangle_R \cap \left\langle \bigcup_{i\in I} Q_i \right\rangle_M \cap \left\langle \bigcup_{i\in I} Q_i \right\rangle_L.$$

By Theorem 2.3, this is obviously a quasi-hyperideal which bounds from above all the quasi-hyperideals $Q_i (i \in I)$. It is the supremum of L. Indeed, for any quasi-hyperideal Q containing all the $Q_i (i \in I)$, we have

$$\vee_{i\in I} Q_i = (\bigcup_{i\in I} Q_i \cup f((\bigcup_{i\in I} Q_i),H,H)) \cap$$
$$(\bigcup_{i\in I} Q_i \cup f(H,(\bigcup_{i\in I} Q_i),H) \cup f(H,H,(\bigcup_{i\in I} Q_i),H,H) \cap$$
$$\cap (\bigcup_{i\in I} Q_i \cup f(H,H,(\bigcup_{i\in I} Q_i))) =$$

$$= \bigcup_{i\in I} Q_i \cup (f((\bigcup_{i\in I} Q_i),H,H) \cap (f(H,(\bigcup_{i\in I} Q_i),H) \cup f(H,H,(\bigcup_{i\in I} Q_i),H,H)) \cap$$

$$\cap f(H,H,(\bigcup_{i\in I} Q_i))) \subseteq f(Q,H,H) \cap (f(H,Q,H) \cup f(H,H,Q,H,H)) \cap$$
$$f(H,H,Q) \subseteq Q.$$

Theorem 2.9 *Let* (H,f) *be a ternary semihypergroup,* e *be an idempotent element,* R *a right hyperideal,* M *a lateral hyperideal and* L *a left hyperideal of* H. *Then* $f(e,e,L), f(R,e,e)$ *and* $f(e,e,M,e,e)$ *are quasi-hyperideals of* H.

Proof. We will show that $f(e,e,L) = L \cap (f(H,e,H) \cup f(H,H,e,H,H)) \cap f(e,H,H)$. Clearly, $f(e,e,L) \subseteq L \cap f(e,H,H)$. If $x \in L \cap f(e,H,H)$

3 ON MINIMAL QUASI-HYPERIDEALS OF TERNARY SEMIHYPERGROUP

Definition 3.1 *Let* (H,f) *be a ternary semihypergroup. A (left, right, lateral, quasi- or bi-) hyperideal* A *of* H *is called to be minimal if it does not properly contain a (left, right, lateral, quasi- or bi-) hyperideal of* H.

Theorem 3.2 *Let* (H,f) *be a ternary semihypergroup. A subset* Q *of* H *is minimal quasi-hyperideal if and only if* M *is the intersection of a minimal left hyperideal, a minimal lateral hyperideal and a minimal right hyperideal of* H.

Proof. Let L be a minimal left hyperideal, M a minimal lateral hyperideal and R a minimal right hyperideal of H. Then by Theorem 2.3, $Q = R \cap M \cap L$ is a quasi-hyperideal. Let us suppose that $Q' \subseteq Q$ be another quasi-hyperideal of H. Then $f(Q',H,H)$ is a right hyperideal and $f(Q',H,H) \subseteq f(Q,H,H) \subseteq R$ and hence $f(Q',H,H) = R$. Similarly $f(Q',H,H) = L$ and $f(H,Q',H) \cup f(H,H,Q',H,H) = M$. Therefore,
$Q = R \cap M \cap L = f(Q',H,H) \cap (f(H,Q',H) \cup f(H,H,Q',H,H)) \cap f(H,H,Q') \subseteq Q'$.
Therefore $Q = Q'$.

Conversely, if Q is a minimal quasi-hyperideal of H, then by the definition

$$f(Q,H,H) \cap (f(H,Q,H) \cup f(H,H,Q,H,H)) \cap f(H,H,Q) \subseteq Q.$$

Let $q \in Q$. Then $f(q,H,H)$ is a right hyperideal, $f(H,q,H) \cup f(H,H,q,H,H)$ is a lateral hyperideal, and $f(H,H,q)$ is a left hyperideal. Hence

$$f(q,H,H) \cap (f(H,q,H) \cup f(H,H,q,H,H)) \cap f(H,H,q)$$

is a quasi-hyperideal contained in Q and therefore by minimality equal to Q. Also we note that $f(q,H,H), f(H,q,H) \cup f(H,H,q,H,H)$, and $f(H,H,q)$ are respectively minimal right, minimal alteral and minimal left hyperideal of H. Let R be any right hyperideal contained in $f(q,H,H)$. Then $f(R,H,H) \subseteq R \subseteq f(q,H,H)$, so that

$$f(R,H,H) \cap (f(H,q,H) \cup f(H,H,q,H,H)) \cap f(H,H,q)$$
$$\subseteq f(q,H,H) \cap (f(H,q,H) \cup f(H,H,q,H,H)) \cap f(H,H,q) = Q.$$

Thus, by minimality of Q, we have $Q = f(R,H,H) \cap (f(H,q,H) \cup f(H,H,q,H,H)) \cap f(H,H,q)$. This means $Q \subseteq f(R,H,H)$. Hence

$$f(R,H,H) \supseteq f(f(R,H,H),H,H) \supseteq f(Q,H,H) \supseteq f(q,H,H).$$

It follows then that $f(q,H,H) = f(R,H,H) = R$. A similar proof holds for the other two cases.

Corollary 3.3 *Let H be a ternary semihypergroup. Any minimal quasi-hyperideal of H is contained in a minimal hyperideal of H.*

Proof. This follows form the fact that any minimal lateral hyperideal is a minimal hyperideal. Let $m \in M$, where M is a minimal lateral hyperideal. Then obviously $f(H,m,H) \cup f(H,H,m,H,H)$ is also a lateral hyperideal of H which is contained in M and hence equal to M by minimality. On the other hand $M = f(H,m,H) \cup f(H,H,m,H,H)$ is also both a left and right hyperideal of H, since

$$f(M,H,H) = f(f(H,m,H),H,H) \cup f(f(H,H,m,H,H),H,H) \subseteq$$
$$\subseteq f(H,m,H) \cup f(H,H,m,H,H)$$
$$f(H,H,M) = f(H,H,f(H,m,H)) \cup f(H,H,f(H,H,m,H,H)) \subseteq$$
$$\subseteq f(H,m,H) \cup f(H,H,m,H,H).$$

4 ON BI-HYPERIDEALS OF TERNARY SEMIHYPERGROUP

Definition 4.1 *A subsemihypergroup B of a ternary semihypergroup H is called a bi-hyperideal of H if $f(B,H,B,H,B) \subseteq B$.*

Lemma 4.2 *Let (H,f) be a ternary semihypergroup. Every quasi-hyperideal of H is a bi-hyperideal of H.*

Remark 4.3 *The converse of the above lemma foes not hold, in general, that is, a bi-hyperideal of a ternary semihypergroup H may not be a quasi-hyperideal of H.*

Remark 4.4 *Since every left, right and lateral hyperideal of H is a quasi-hyperideal of H, it follows that every left, right and lateral hyperideal of H is a bi-hyperideal of H, bu the converse is not true, in general.*

Proposition 4.5 *Let (H,f) be a ternary semihypergroup. If B is a bi-hyperideal of H and T is a ternary sub-semihypergroup of H, then $B \cap T$ is a bi-hyperideal of T.*

Lemma 4.6 *Let (H,f) be a ternary semihypergroup. If B is a bi-hyperideal of a ternary semihypergroup H and T_1,T_2 are two ternary sub-semihypergroup of H, then $f(B,T_1,T_2), f(T_1,B,T_2)$ and $f(T_1,T_2,B)$ are bi-hyperideal of H.*

Corollary 4.7 *Let (H,f) be a ternary semihypergroup. If B_1,B_2 and B_3 are three bi-hyperideals of a ternary semihypergroup H, then $f(B_1,B_2,B_3)$ is a bi-hyperideal of H.*

Corollary 4.8 *Let (H,f) be a ternary semihypergroup. If Q_1,Q_2 and Q_3 are three quasi-hyperideals of a ternary semihypergroup H, then $f(Q_1,Q_2,Q_3)$ is a bi-hyperideal of H.*

In general, if B is a bi-hyperideal of a ternary semihypergroup H and C is a bi-hyperideal of B, then C is not a bi-hyperideal of H. But, in particular, we have the following result.

Theorem 4.9 *Let* (H,f) *be a ternary semihypergroup,* B *be a bi-hyperideal of* H *and* C *be a bi-hyperideal of* B *such that* $f(C,C,C)=C$. *Then* C *is a bi-hyperideal of* H.

Proposition 4.10 *Let* (H,f) *be a ternary semihypergroup. Let* X,Y,Z *be three ternary sub-semihypergroups of a* H *and* $B=f(X,Y,Z)$. *Then,* B *is a bi-hyperideal if at least one of* X,Y,Z *is a right, a lateral, or a left hyperideal of* H.

Corollary 4.11 *Let* (H,f) *be a ternary semihypergroup. A ternary sub-semihypergroup* B *of* H *is a bi-hyperideal of* H *if* $B=f(R,M,L)$, *where* R *is a right hyperideal,* M *is a lateral hyperideal and* L *is a left hyperideal of* H.

Proposition 4.12 *Let* (H,f) *be a ternary semihypergroup and* B *a ternary sub-semihypergroup of* H. *If* R *is a right yperideal,* M *is a lateral hyperideal and* L *is a left hyperideal of* H *such that* $f(R,M,L)\subseteq B\subseteq R\cap M\cap L$, *then* B *is a bi-hyperideal of* H.

Proposition 4.13 *Let* (H,f) *be a ternary semihypergroup,* A *be a hyperideal of* H *and* Q *be a quasi-hyperideal of* H. *Then* $A\cap Q$ *is a bi-hyperideal and a quasi-hyperideal of* H.

Proposition 4.14 *Let* (H,f) *be a ternary semihypergroup. The intersection of arbitrary set of bi-hyperideals of* H *is either empty or a bi-hyperideal of* H.

5 ON PRIME, STRONGLY PRIME AND SEMIPRIME BI-HYPERIDEALS OF TERNARY SEMIHYPERGROUP

Throughout this section we will consider H as a ternary semihypergroup with zero.

Definition 5.1 *Let* (H,f) *be a ternary semihypergroup. A bi-hyperideal* B *of* H *is called*

1. *prime* if $f(B_1,B_2,B_3)\subseteq B\Rightarrow B_1\subseteq B$ or $B_2\subseteq B$ or $B_3\subseteq B$ for any bi-hyperideals B_1,B_2,B_3 of H.
2. *strongly prime* if $f(B_1,B_2,B_3)\cap f(B_2,B_3,B-1)\cap f(B_3,B_1,B_2)\subseteq B\Rightarrow$ $B_1\subseteq B$ or $B_2\subseteq B$ or $B_3\subseteq B$ for any bi-hyperideals B_1,B_2,B_3 of H.
3. *semiprime* if $f(B_1,B_1,B_1)\subseteq B\Rightarrow B_1\subseteq B$ for any bi-hyperideal B_1 of H.

Remark 5.2 *Every strongly prime bi-hyperideal of a ternary semihypergroup* H *is a prime bi-hyperideal and every prime bi-hyperideal is a semiprime bi-hyperideal. A prime bi-hyperideal is not necessarily a strong prime bi-hyperideal and a semiprime bi-hyperideal is not necessarily a prime bi-hyperideal.*

Example 5.3 *Let* $H=\{0,a,b,c,d,e,g\}$ *and* $f(x,y,z)=(x*y)*z$ *for all* $x,y,z\in H$, *where* $*$ *is defined by the table:*

$*$	0	a	b	c	d	e	g
0	0	0	0	0	0	0	0
a	0	a	$\{a,b\}$	c	$\{c,d\}$	e	$\{e,g\}$
b	0	b	b	d	d	g	g
c	0	c	$\{c,d\}$	c	$\{c,d\}$	c	$\{c,d\}$
d	0	d	d	d	d	d	d
e	0	e	$\{e,g\}$	c	$\{c,d\}$	e	$\{e,g\}$
g	0	g	g	d	d	g	g

Then (H,f) is a ternary semihypergroup. Clearly, $B_1=\{0\},B_2=\{0,c,d\}$, $B_3=\{0,c,d,e,g\}$, $B_4=\{0,b,d,g\}$ are bi-hyperideals of H. All bi-hyperideals are prime and hence semiprime. The prime bi-hyperideal $\{0\}$ is not strongly prime bi-hyperideal.

It is easily to verify the that the following proposition is true.

Proposition 5.4 *Let* H *be a ternary semihypergroup. The intersection of any family of prime bi-hyperideals of* H *is a semiprime bi-hyperideal.*

6. IRREDUCIBLE AND STRONGLY IRREDUCIBLE BI-HYPERIDEALS

Definition 6.1 *Let* (H,f) *be a ternary semihypergroup. A bi-hyperideal* B *of* H *is called*

1. *irreducible* if $B_1 \cap B_2 \cap B_3 = B \Rightarrow B_1 = B$ *or* $B_2 = B$ *or* $B_3 = B$ *for any bi-hyperideals* B_1, B_2, B_3 *of* H.
2. *strongly irreducible* if $B_1 \cap B_2 \cap B_3 \subseteq B \Rightarrow B_1 \subseteq B$ *or* $B_2 \subseteq B$ *or* $B_3 \subseteq B$ *for any bi-hyperideals* B_1, B_2, B_3 *of* H.

Every strongly irreducible bi-hyperideal of a ternary semihypergroup H is an irreducible bi-hyperideal but the converse is not true in general.
In the Example 5.3, all the bi-hyperideals are irreducible bi-hyperideals. Strongly irreducible bi-hyperideals are only B_2, B_3, B_4.

Proposition 6.2 *Let* H *be a ternary semihypergroup. Every strongly irreducible semiprime bi-hyperideal of* H *is strongly prime.*
Proof. Let B be a strongly irreducible semiprime bi-hyperideal of H. Let us suppose that B_1, B_2 and B_3 are bi-hyperideals of H such that

$$f(B_1,B_2,B_3) \cap f(B_2,B_3,B_1) \cap f(B_3,B_1,B_2) \subseteq B.$$

Since

$$f(B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3) \subseteq f(B_1,B_2,B_3)$$

$$f(B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3) \subseteq f(B_2,B_3,B_1)$$
$$f(B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3) \subseteq f(B_3,B_1,B_2)$$

we have

$$f(B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3) \subseteq f(B_1,B_2,B_3) \cap f(B_2,B_3,B_1) \cap f(B_3,B_1,B_2) \subseteq B.$$

But B is semiprime, so $f(B_1 \cap B_2 \cap B_3) \subseteq B$.

Since B is strongly irreducible, we have either $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$. Thus B is stongly prime bi-hyperideal of H.

Proposition 6.3 *Let* H *be a ternary semihypergroup and* B *be a bi-hyperideal of* H. *For any* $a \in H \setminus B$ *there exists an irreducible bi-hyperideal* I *of* H *such that* $B \subseteq I$ *and* $a \notin I$.

Proof. Let us suppose $F = \{B_i : i \in I\}$ be the collection of all bi-hyperideals of H which contains B and does not contain a, then $F \neq \varnothing$ because $B \in F$. Evidently F is partially ordered under inclusion. If Ω is any totally ordered subset of F, then $\bigcup \Omega$ is a bi-hyperideal of H containing B and not containing a. Hence by Zorn's lemma, there exists a maximal element I in F. We show that I is an irreducible bi-hyperideal of H. Let C, D and E be any three bi-hyperideals of H such that $I = C \cap D \cap E$. If all of three bi-hyperideals C, D and E properly contain I, then according to the hypothesis $a \in C, a \in D$ and $a \in E$. Hence $a \in C \cap D \cap E = I$. This contradicts the fact that $a \notin I$. Thus either $I = C$ or $I = D$ or $I = E$. Hence I is irreducible.

Proposition 6.4 *Let* H *be a regular ternary semihypergroup and* B *be a bi-hyperideal of* H, T_1, T_2 *are non-empty subsets of* H. *Then* $f(B,T_1,T_2), f(T_1,T_2,B)$ *are bi-hyperideals of* H.
Proof. Let H be a regular ternary semihypergroup, B a bi-hyperideal of H and T_1, T_2 are non-empty subsets of H. Then,

$$\begin{aligned}
&f(f(B,T_1,T_2), f(B,T_1,T_2), f(B,T_1,T_2)) \\
&\subseteq f(B, f(T_1,T_2,B), f(T_1,T_2,B), T_1, T_2) \\
&\subseteq f(B, f(H,H,B), f(H,H,B), T_1, T_2) \\
&= f(B, f(H,H,B,H,H), f(B,T_1,T_2)) \\
&\subseteq f(B, f(H,H,H,H,H), B, T_1, T_2) \\
&\subseteq f(B, f(H,H,H), B, T_1, T_2) \\
&\subseteq f(B,H,B), T_1, T_2) = f(B,T_1,T_2).
\end{aligned}$$

because in a regular ternary semigroup $B = f(B,H,B)$. Thus $f(B,T_1,T_2)$ is a ternary subsemihypergroup of H. Also

$$\begin{aligned}
&f(f(B,T_1,T_2), H, f(B,T_1,T_2), H, f(B,T_1,T_2)) \\
&\subseteq f(B, f(T_1,T_2,H), B, f(T_1,T_2,H), B, T_1, T_2) \\
&\subseteq f(B, f(H,H,H), B, f(H,H,H), B, T_1, T_2) \\
&\subseteq f(f(B,H,B,H,B), T_1, T_2) \subseteq f(B,T_1,T_2).
\end{aligned}$$

Hence $f(B,T_1,T_2)$ is a bi-hyperideal of H.

Similarly, we can show that $f(T_1,B,T_2)$, $f(T_1,T_2,B)$ are bi-hyperideals of H.

Corollary 6.5 *If* B_1, B_2 *and* B_3 *are bi-hyperideals of a regular ternary semihypergroup* H, *then* $f(B_1, B_2, B_3)$ *is a bi-hyperideal of* H.

Corollary 6.6 *If* Q_1, Q_2, Q_3 *are quasi-hyperideals of a regular ternary semihypergroup* H, *then* $f(Q_1, Q_2, Q_3)$ *is a bi-hyperideal.*

Theorem 6.7 *Let* H *be a regular ternary semihypergroup. The following assertions are equivalent:*

1. *every bi-hyperideal of* H *is idempotent*,
2. $B_1 \cap B_2 \cap B_3 = f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2)$ *for every bi-hyperideals of* H,
3. *every bi-hyperideal of* H *is semiprime,*
4. *each proper bi-hyperideal of* H *is the intersection of all irreducible semiprime bi-hyperideals of* H *which contain it*.

Proof. $(1) \Rightarrow (2)$. Let B_1, B_2 and B_3 be bi-hyperideals of H. Then by the hypothesis

$$B_1 \cap B_2 \cap B_3 = f(B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3, B_1 \cap B_2 \cap B_3) \subseteq f(B_1, B_2, B_3).$$

Similarly,

$$B_1 \cap B_2 \cap B_3 \subseteq f(B_2, B_3, B_1) \text{ and}$$
$$B_1 \cap B_2 \cap B_3 \subseteq f(B_3, B_1, B_2).$$

Thus

$$B_1 \cap B_2 \cap B_3 \subseteq f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2). \tag{1}$$

By Corollary 6.5, $f(B_1, B_2, B_3), f(B_2, B_3, B_1), f(B_3, B_1, B_2)$ are bi-hyperideals. By Lemma 2.7, $f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2)$ is a bi-hyperideal. Thus by hypothesis $f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2) =$

$$= f(f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2), f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1)$$
$$\cap f(B_3, B_1, B_2), f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2))$$
$$\subseteq f(f(B_1, B_2, B_3), f(B_2, B_3, B_1), f(B_3, B_1, B_2))$$
$$\subseteq f(f(B_1, H, H), f(H, B_1, H), f(H, H, B_1))$$
$$= f(B_1, f(H, H, H), B_1, f(H, H, H), B_1) \subseteq f(B_1, H, B_1, H, B_1) \subseteq B_1.$$

Similarly,

$$f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2) \subseteq B_2$$

and

$$f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2) \subseteq B_3.$$

Thus

$$f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2) \subseteq B_1 \cap B_2 \cap B_3. \tag{2}$$

From (1) and (2) we have

$$B_1 \cap B_2 \cap B_3 = f(B_1, B_2, B_3) \cap f(B_2, B_3, B_1) \cap f(B_3, B_1, B_2).$$

$(2) \Rightarrow (1)$. It is obvious.

$(1) \Rightarrow (3)$. Let B and B_1 by any two bi-hyperideals of H such that $f(B_1, B_1, B_1) \subseteq B$. Then by hypothesis, $B_1 = f(B_1, B_1, B_1) \subseteq B$. Hence every bi-hyperideal of H is semiprime.

$(3) \Rightarrow (4)$. Let B be a proper of H, then B is contained in the intersection of all irreducible bi-hyperideals of H which contain B. Proposition 6.3 guarantees the existence of such irreducible bi-hyperideals. If $a \notin B$, then there exists an irreducible bi-hyperideal of H which contains B but does not contain a. Thus B is the intersection of all irreducible bi-hyperideals of H which contain B. By hypothesis, each bi-hyperideal is semiprime, so each bi-hyperideal is the intersection of irreducible semiprime bi-hyperideals of H which contains it.

$(4) \Rightarrow (1)$. Let B be a bi-hyperideal of a ternary semihypergroup H. If $f(B,B,B) = H$, then clearly B is idempotent. If $f(B,B,B) \neq H$, then $f(B,B,B)$ is a proper bi-hyperideal of H containing $f(B,B,B)$, so by the hypothesis,

$$f(B,B,B) = \begin{array}{l}\{B_\alpha : B_\alpha \text{ is irreducible semiprime bi-} \\ \text{hyperideal of } H \text{ containing } f(B,B,B)\}.\end{array}$$

Since each B_α is semiprime bi-hyperideal, $f(B,B,B) \subseteq B_\alpha$ implies $B \subseteq B_\alpha$. Therefore $B \subseteq \bigcap B_\alpha = f(B,B,B)$ implies $B \subseteq f(B,B,B)$, but $f(B,B,B) \subseteq B$. Hence $f(B,B,B) = B$.

Proposition 6.8 *Let* H *be a ternary semihypergroup. If each bi-hyperideal of* H *is idempotent, then a bi-hyperideal* B *of* H *is strongly irreducible if and only if* B *is strongly prime.*

Proof. Let us suppose that a bi-hyperideal B is strongly irreducible and let B_1, B_2, B_3 are bi-hyperideals of H such that $f(B_1,B_2,B_3) \cap f(B_2,B_3,B_1) \cap f(B_3,B_1,B_2) \subseteq B$.

By Theorem 6.7, we have $B_1 \cap B_2 \cap B_3 = f(B_1,B_2,B_3) \cap f(B_2,B_3,B_1) \cap f(B_3,B_1,B_2)$, so we have $B_1 \cap B_2 \cap B_3 \subseteq B$. Since B is strongly irreducible so, either $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$. Thus B is strongly prime. On the other hand, if B is strongly prime and $B_1 \cap B_2 \cap B_3 \subseteq B$ for some bi-hyperideals B_1, B_2 and B_3 of H, then, by Theorem 6.7, we have

$$f(B_1,B_2,B_3) \cap f(B_2,B_3,B_1) \cap f(B_3,B_1,B_2) \subseteq B,$$

whenece we conclude either $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$. Therefore B is strongly irreducible.

Next we characterize those ternary semihypergroups for which each bi-hyperideal is strongly irreducible and also those ternary semihypergroups in which each bi-hyperideal is strongly prime.

Theorem 6.9 *Let* H *be a regular ternary semihypergroup. Each bi-hyperideal of* H *is strongly prime if and only if every bi-hyperideal of* H *is idempotent and the set of bi-hyperideals of* H *is totally ordered by inclusion.*

Proof. Let us suppose that each bi-hyperideal of H is strongly prime, then each bi-hyperideal of H is semiprime. Thus by Theorem 6.7, every bi-hyperideal of H is idempotent. We show that the set of bi-hyperideals of H is totally ordered by inclusion. Let B_1 and B_2 be any two bi-hyperideals of H, then by Theorem 6.7,

$$B_1 \cap B_2 = B_1 \cap B_2 \cap H = f(B_1,B_2,H) \cap f(B_2,H,B_1) \cap f(H,B_1,B_2).$$

Thus

$$f(B_1,B_2,H) \cap f(B_2,H,B_1) \cap f(H,B_1,B_2) \subseteq B_1 \cap B_2.$$

As each bi-hyperideal is strongly prime, therefore $B_1 \cap B_2$ is strongly prime bi-hyperideal. Hence either $B_1 \subseteq B_1 \cap B_2$ or $B_2 \subseteq B_1 \cap B_2$ or $H \subseteq B_1 \cap B_2$. Now, if $B_1 \subseteq B_1 \cap B_2$, then $B_1 \subseteq B_2$; if $B_2 \subseteq B_1 \cap B_2$, then $B_2 \subseteq B_1$; if $H \subseteq B_1 \cap B_2$, then $B_1 = H = B_2$. Thus set of bi-hyperideals of H is totally ordered under inclusion.

Conversely, assume that every bi-hyperideal of H is idempotent and the set of bi-hyperideals of H is totally ordered under inclusion. We show that each bi-ideal of H is strongly prime. Let B, B_1, B_2 and B_3 be bi-hyperideals of H such that

$$f(B_1,B_2,B_3) \cap f(B_2,B_3,B_1) \cap f(B_3,B_1,B_2) \subseteq B.$$

Since every bi-hyperideal of H is idempotent, by Theorem 6.7, $B_1 \cap B_2 \cap B_3 \subseteq B$.

Since the set of all bi-hyperideals of H is totally ordered under inclusion so for B_1, B_2, B_3 we have the following six possibilities:

(1) $B_1 \subseteq B_2; B_2 \subseteq B_3; B_1 \subseteq B_3$, (2) $B_1 \subseteq B_2; B_3 \subseteq B_2; B_1 \subseteq B_3$,

(3) $B_1 \subseteq B_2; B_3 \subseteq B_2; B_3 \subseteq B_1$, (4) $B_2 \subseteq B_1; B_2 \subseteq B_3; B_1 \subseteq B_3$,

(5) $B_2 \subseteq B_1; B_3 \subseteq B_2; B_3 \subseteq B_1$, (6) $B_2 \subseteq B_1; B_3 \subseteq B_1; B_2 \subseteq B_3$.

In these cases we have

(1) $B_1 \cap B_2 \cap B_3 = B1$, (2) $B_1 \cap B_2 \cap B_3 = B_1$; (3) $B_1 \cap B_2 \cap B_3 = B_3$,

(4) $B_1 \cap B_2 \cap B_3 = B_2$, (5) $B_1 \cap B_2 \cap B_3 = B_3$, (6) $B_1 \cap B_2 \cap B_3 = B_2$.

Thus either $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$, which proves that B is strongly prime.

Theorem 6.10 *Let* H *be a regular ternary semihypergroup. If the set of bi-hyperideals of* H *is totally ordered, then every bi-hyperideal of* H *is idempotent if and only if each bi-hyperideal of* H *is prime.*

Proof. Let us suppose every bi-hyperideal of H is idempotent. Let B, B_1, B_2, B_3 be bi-hyperideals of H such that $f(B_1,B_2,B_3) \subseteq B$.

As in the proof of the previous theorem we obtain $B_1 \subseteq B_2, B_2 \subseteq B_3, B_1 \subseteq B_3$, whence we conclude $f(B_1,B_1,B_1) \subseteq f(B_1,B_2,B_3) \subseteq B$, i.e., $f(B_1,B_1,B_1) \subseteq B$. By Theorem 6.7, B is a semiprime bi-hyperideal, so $B_1 \subseteq B$. Similarly for other cases we have $B_2 \subseteq B$ or $B_3 \subseteq B$.

Conversely, assume that every bi-hyperideal of H is prime. Since the set of bi-hyperideals of H is totally ordered under inclusion, so the concepts of primeness and strongly primeness coincide. Hence by Theorem 6.7, every bi-hyperideal of H is idempotent.

Theorem 6.11 *Let* H *be a ternary semihypergroup. The following statements are equivalent:*

1. *the set of bi-hyperideals of* H *is totally ordered under inclusion*,
2. *each bi-hyperideal of* H *is strongly irreducible,*
3. *each bi-hyperideal of* H *is irreducible.*

Proof. $(1) \Rightarrow (2)$. Let $B_1 \cap B_2 \cap B_3 \subseteq B$ for some bi-hyperideals of H. Since the set of bi-hyperideals of H is totally ordered under inclusion, therefore either $B_1 \cap B_2 \cap B_3 = B_1$ or B_2 or B_3. Thus either $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$. Hence B is strongly irreducible.

$(2) \Rightarrow (3)$. If $B_1 \cap B_2 \cap B_3 = B$ for some bi-hyperideals of H, then $B \subseteq B_1$, $B \subseteq B_2$ and $B \subseteq B_3$. On the other hand by hypothesis we have, $B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$. Thus $B_1 = B$ or $B_2 = B$ or $B_3 = B$. Hence B is irreducible.

$(3) \Rightarrow (1)$. Let us suppose each bi-hyperideal of H is irreducible. Let B_1, B_2 be bi-hyperideals of H, then $B_1 \cap B_2$ is also a bi-hyperideal of H. Since $B_1 \cap B_2 \cap H = B_1 \cap B_2$, the irreducibility of $B_1 \cap B_2$ implies that either $B_1 = B_1 \cap B_2$ or $B_2 = B_1 \cap B_2$ or $H = B_1 \cap B_2$, i.e., either $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$ or $B_1 = B_2$. Hence the set of bi-hyperideals of H is totally ordered under inclusion.

Let $\mathcal{B}$ be the family of all bi-hyperideals of H and $\mathcal{P}$ the family of all proper strongly prime bi-hyperideals of H. For each $B \in \mathcal{B}$ we define

$$\Theta_B = \{J \in \mathcal{P} : B \nsubseteq J\} \text{ and}$$

$F(\mathcal{P}) = \{\Theta_B : B \text{ is a bi-hyperideal of } H\}$.

Theorem 6.12 *If* H *is a ternary semihypergroup with the property that every bi-hyperideal of* H *is idempotent, then* $F(\mathcal{P})$ *forms a topology on the set* $\mathcal{P}$.

Proof. As $\{0\}$ is a bi-hyperideal of H, so $\Theta_0 = \{J \in \mathcal{P} : \{0\} \nsubseteq J\} = \varnothing$ because 0 belong to every bi-hyperideal. Since H is a bi-hyperideal of H, we have $\Theta_H = \{J \in \mathcal{P} : H \nsubseteq J\} = \mathcal{P}$ because $\mathcal{P}$ is the collection of all proper strongly prime bi-hyperideals in H. Thus $\varnothing$ and $\mathcal{P}$ belongs to $F(\mathcal{P})$. Let $\{\Theta_{B_\alpha} : \alpha \in I\} \subseteq F(\mathcal{P})$. Then

$$\bigcup_{\alpha\in I} \Theta_{B_\alpha} = \{J \in \mathcal{P} : B_\alpha \nsubseteq J \text{ for some } \alpha \in I\} = \{J \in \mathcal{P} : \bigcup_{\alpha\in I} B_\alpha \nsubseteq J\}$$

which is equal to $\Theta_{\bigcup_{\alpha\in I} B_\alpha} \in F(\mathcal{P})$, where $\bigcup_{\alpha\in I} B_\alpha$ means the bi-hyperideal of H generated by $\bigcup_{\alpha\in I} B_\alpha$.

Let Θ_{B_1} and Θ_{B_2} be arbitrary two elements from $F(\mathcal{P})$. We show that $\Theta_{B_1} \cap \Theta_{B_2} \in F(\mathcal{P})$. If $J \in \Theta_{B_1} \cap \Theta_{B_2}$, then $J \in \mathcal{P}, B_1 \nsubseteq J$ and $B_2 \nsubseteq J$. Let us suppose that $B_1 \cap B_2 = B_1 \cap B_2 \cap H \subseteq J$. By Theorem 6.7, we have $f(B_1,B_2,H) \cap f(B_2,H,B_1) \cap f(H,B_1,B_2) \subseteq J$. Since J is a strongly prime bi-hyperideal, therefore either $B_1 \subseteq J$ or $B_2 \subseteq J$ ($H \nsubseteq J$ because J is a proper bi-hyperideal of H), which is a contradiction. Hence $B_1 \cap B_2 \nsubseteq J$, i.e., $J \in \Theta_{B_1 \cap B_2}$. Thus $\Theta_{B_1} \cap \Theta_{B_2} \subseteq \Theta_{B_1 \cap B_2}$.

On the other hand, if $J \in \Theta_{B_1 \cap B_2}$, then $J \in \mathcal{P}$ and $B_1 \cap B_2 \nsubseteq J$, which means that $B_1 \nsubseteq J$ and $B_2 \nsubseteq J$. Therefore, $J \in \Theta_{B_1}$ and $J \in \Theta_{B_2}$, i.e.,

$J \in \Theta_{B_1} \cap \Theta_{B_2}$. Hence $\Theta_{B_1 \cap B_2} \subseteq \Theta_{B_1} \cap \Theta_{B_2}$. Thus $\Theta_{B_1 \cap B_2} = \Theta_{B_1} \cap \Theta_{B_2}$, so $\Theta_{B_1} \cap \Theta_{B_2} \in F(P)$. This proves that $F(P)$ is a topology on P.

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